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NEW OPERATORS OVER THE GENERALIZED INTERVAL VALUED INTUITIONISTIC FUZZY SETS

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Abstract

In this paper, newly defined four operators over generalized interval valued intuitionistic fuzzy sets are proposed. Some of the basic properties of the new operators are discussed.

1. Introduction

In recent decades, several types of sets, such as fuzzy sets (FS) (Zadeh [30]), interval valued fuzzy sets (IVFS) (Zadeh [31]), intuitionistic fuzzy sets (IFS) (Atanassov [1]), intuitionistic fuzzy sets of root type (Srinivasan and Palaniappan [19]), intuitionistic fuzzy sets of second type (Atanassov [4]), interval valued intuitionistic fuzzy sets (IVIFS)

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(Atanassov and Gargov [2]), type-2 fuzzy sets (John [13]), type-n fuzzy sets (Dubois and Prade [11]), fuzzy multisets (Yager [29]), vague sets (Gau and Buehrer [12]) hesitant fuzzy sets (Torra and Narukawa [23]), generalized interval valued intuitionistic fuzzy sets (GIVIFS) (Bhowmik and Pal [8, 9]) have been introduced and investigated widely for modelling several real life problems. Atanassov [3] defined different operators over IVIFS. Xu and Jian [26] and Xu [27, 28] developed some arithmetic aggregation operators and some geometric aggregation operators of IVIFS for decision making. Li [14, 15, 16], Chen et al. [10], Sahin [18], and Liu and Luo [17] presented methods for multi-criteria fuzzy decision making based on IVIFS. Bhowmik and Pal [7] define two operators C and I with some properties over GIVIFSs. Wang et al. [25] defined two new aggregation operators based on the Łukasiewicz triangular norm. Wang and Liu [24] considered the interval valued intuitionistic fuzzy hybrid weighted averaging operator based on Einstein operation and its application to decision making. Sudharsan and Ezhilmaran [20] defined two new operators over IVIFSs. Sudharsan and Ezhilmaran [21] proposed two new operators defined over IFSs and also two new operators defined over an IVIFS. Sudharsan and Ezhilmaran [22] present a weighted arithmetic average operator based on interval valued intuitionistic fuzzy values and their application to multi-criteria decision making for investment.

Baloui Jamkhaneh and Nadarajah [5] considered a new generalized intuitionistic fuzzy sets (GIFS_B) and introduced some operators over GIFS_B. Baloui Jamkhaneh [6] considered new generalized interval valued intuitionistic fuzzy sets (GIVIFS_B) and introduced some operators over GIVIFS_B. In this paper, our aim is to propose four new operators on GIVIFS_Bs and study their properties.

2. Preliminaries

In this section, we give some basic definition. Let X be a non-empty set.

Definition 2.1 (Atanassov [1]). An IFS A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$ denote the degree of membership and non-membership functions of A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2. Let $[I]$ be the set of all closed subintervals of the interval $[0, 1]$ and $M_A(x) = [M_{AL}(x), M_{AU}(x)] \in [I]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)] \in [I]$ then $N_A(x) \leq M_A(x)$ if and only if $N_{AL}(x) \leq M_{AL}(x)$ and $N_{AU}(x) \leq M_{AU}(x)$.

Definition 2.3 (Atanassov & Gargov [2]). Interval valued intuitionistic fuzzy sets (IVIFS) A in X, is defined as an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$, where the functions $M_A(x) : X \to [I]$ and $N_A(x) : X \to [I]$, denote the degree of membership and degree of non-membership of A, respectively, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$, $N_A(x) = [N_{AL}(x), N_{AU}(x)]$, $0 \leq M_{AU}(x) + N_{AU}(x) \leq 1$ for each $x \in X$.

Definition 2.4 (Baloui Jamkhaneh and Nadarajah [5]). Generalized intuitionistic fuzzy sets $(\mathrm{GIFS_B})$ A in X, is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$, denote the degree of membership and degree of non-membership functions of A, respectively, and $0 \leq \mu_A(x)^{\delta} + \upsilon_A(x)^{\delta} \leq 1$ for each $x \in X$ and $\delta = n$ or $\frac{1}{n}$, $n = 1, 2, \ldots, N$.

Definition 2.5 (Baloui Jamkhaneh [6]). Generalized interval valued intuitionistic fuzzy sets $(\mathrm{GIVIFS_B})$ A in X, is defined as an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$, where the functions $M_A(x) : X \to [I]$ and $N_A(x) : X \to [I]$, denote the degree of membership and degree of non-membership of A, respectively, and $M_A(x) = [M_{AL}(x), M_{AU}(x)]$, $N_A(x) = [N_{AL}(x), N_{AU}(x)]$, where

$0 \leq M_{AU}(x)^{\delta} + N_{AU}(x)^{\delta} \leq 1,$ for each $x \in X$ and $\delta = n$ or $\frac{1}{n}$, $n = 1, 2, \ldots, N$. The collection of all $GIVIFS_B(\delta)$ is denoted by $GIVIFS_B(\delta, X)$.

Definition 2.6 (Baloui Jamkhaneh [6]). Let A and B be two $GIVIFS_Bs$ such that

$$A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}, B = \{\langle x, M_B(x), N_B(x)\rangle : x \in X\},$$

$$M_A(x) = [M_{AL}(x), M_{AU}(x)], \quad N_A(x) = [N_{AL}(x), N_{AU}(x)],$$

$$M_B(x) = [M_{BL}(x), M_{BU}(x)], N_B(x) = [N_{BL}(x), N_{BU}(x)].$$

Define the following relations on A and B:

(i) $A \subset B$ if and only if $M_A(x) \leq M_B(x)$ and $N_A(x) \geq N_B(x)$, $\forall x \in X$;

(ii) $A \cup B = \{\langle x, [\max(M_{AL}(x), M_{BL}(x)), \max(M_{AU}(x), M_{BU}(x))],$

$[\min(N_{AL}(x), N_{BL}(x)), \min(N_{AU}(x), N_{BU}(x))]\rangle : x \in X\};$

(iii) $A \cap B = \{\langle x, [\min(M_{AL}(x), M_{BL}(x)), \min(M_{AU}(x), M_{BU}(x))],$

$[\max(N_{AL}(x), N_{BL}(x)), \max(N_{AU}(x), N_{BU}(x))]\rangle : x \in X\};$

(iv) $\overline{A} = \{\langle x, N_A(x), M_A(x)\rangle : x \in X\}.$

Definition 2.7. For every $GIVIFS_B A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$, we define the modal logic operators "necessity" and "possibility".

The Necessity measure on A:

$$\square A = \left\{\langle x, [M_{AL}(x), M_{AU}(x)], [N_{AL}(x), (1 - M_{AU}(x)^{\delta})^{\frac{1}{\delta}}]\rangle : x \in X\right\}.$$

The Possibility measure on A:

$$\Diamond A = \left\{\langle x, [M_{AL}(x), (1 - N_{AU}(x)^{\delta})^{\frac{1}{\delta}}], [N_{AL}(x), N_{AU}(x)]\rangle : x \in X\right\}.$$

Corollary 2.1. *Let* $A, B \in GIVIFS_B$, *we have*

(i) $\Box A \in GIVIFS_B$,

(ii) $\Diamond A \in GIVIFS_B$,

(iii) $\Box(A \cup B) = \Box A \cup \Box B$,

(iv) $\Diamond(A \cup B) = \Diamond A \cup \Diamond B$,

(v) $\Box(A \cap B) = \Box A \cap \Box B$,

(vi) $\Diamond(A \cap B) = \Diamond A \cap \Diamond B$.

Corollary 2.2. *Let* $A, B \in GIVIFS_B$, *we have*

(i) $\Diamond \Box A = \Box A$,

(ii) $\Box \Diamond A = \Diamond A$,

(iii) $\Box \overline{A} = \overline{\Diamond A}$,

(iv) $\Diamond \overline{A} = \overline{\Box A}$.

Corollary 2.3. *Let* $A, B \in GIVIFS_B$, $A \subset B$, *we have*

(i) $\Box A \subset \Box B$,

(ii) $\Diamond A \subset \Diamond B$.

3. The Operators of $\mathrm{GIVIFS_B}$

Here, we will introduce new operators over the $\mathrm{GIVIFS_B}$, which extend some operators in the research literature related to IVIFSs. Let X is a non-empty finite set and $A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$ is a $\mathrm{GIVIFS_B}$.

Definition 3.1. Let $\alpha, \beta \in [0, 1]$ and $A \in GIVIFS_B$, we define the operator of $J^*_{\alpha,\beta}(A)$ as follows:

$$J^*_{\alpha,\beta}(A) = \left\{\langle x,\ M_{J^*_{\alpha,\beta}}(A),\ N_{J^*_{\alpha,\beta}}(A)\rangle : x \in X\right\},$$

$$M_{J^*_{\alpha,\beta}}(A) = \left[M_{AL}(x),\ \left(M_{AU}(x)^{\delta} + \alpha(1 - M_{AU}(x)^{\delta} - \beta N_{AU}(x)^{\delta})\right)^{\frac{1}{\delta}}\right],$$

$$N_{J^*_{\alpha,\beta}}(A) = [\beta^{\frac{1}{\delta}} N_{AL}(x),\ \beta^{\frac{1}{\delta}} N_{AU}(x)].$$

Theorem 3.1. *For every* $A \in GIVIFS_B$, *and for every three real numbers* $\alpha, \beta, \gamma \in [0, 1]$

(i) $J^*_{\alpha,\beta}(A) \in GIVIFS_B$,

(ii) $\alpha \leq \gamma \Rightarrow J^*_{\alpha,\beta}(A) \subset J^*_{\gamma,\beta}(A)$,

(iii) $\beta \leq \gamma \Rightarrow J^*_{\alpha,\beta}(A) \supset J^*_{\alpha,\gamma}(A)$,

(iv) $J^*_{1,1}(A) = \Diamond A$,

(v) $J^*_{0,1}(A) = A$.

Proof. (i)

$$M_{J^*_{\alpha,\beta}(A)U}(x)^{\delta} + N_{J^*_{\alpha,\beta}(A)U}(x)^{\delta}$$

$$= \left(\left(M_{AU}(x)^{\delta} + \alpha(1 - M_{AU}(x)^{\delta} - \beta N_{AU}(x)^{\delta})\right)^{\frac{1}{\delta}}\right)^{\delta} + \left(\beta^{\frac{1}{\delta}} N_{AU}(x)\right)^{\delta}$$

$$= \left(M_{AU}(x)^{\delta} + \alpha(1 - M_{AU}(x)^{\delta} - \beta N_{AU}(x)^{\delta}\right) + \beta N_{AU}(x)^{\delta}$$

$$\leq M_{AU}(x)^{\delta} + 1 - M_{AU}(x)^{\delta} - \beta N_{AU}(x)^{\delta} + \beta N_{AU}(x)^{\delta} = 1.$$

Finally, it can be concluded that $J^*_{\alpha,\beta}(A) \in GIVIFS_B$.

(ii) Since $\alpha \leq \gamma$, then it is clear that

$$\left[M_{AL}(x),\ \left(M_{AU}(x)^{\delta} + \alpha(1 - M_{AU}(x)^{\delta} - \beta N_{AU}(x)^{\delta})\right)^{\frac{1}{\delta}}\right]$$

$$\leq \left[M_{AL}(x),\ \left(M_{AU}(x)^{\delta} + \gamma(1 - M_{AU}(x)^{\delta} - \beta N_{AU}(x)^{\delta})\right)^{\frac{1}{\delta}}\right].$$

Finally, we have $J^{*}_{\alpha,\beta}(A) \subset J^{*}_{\gamma,\beta}(A)$.

The proof of (iii) is similar to that of (ii). Proofs (iv) and (v) are obvious.

Definition 3.2. Let $\alpha, \beta \in [0, 1]$ and $A \in GIVIFS_B$, we define the operator of $j^{*}_{\alpha,\beta}(A)$ as follows:

$$j^{*}_{\alpha,\beta}(A) = \left\{\langle x,\ M_{j^{*}_{\alpha,\beta}}(A),\ N_{j^{*}_{\alpha,\beta}}(A)\rangle : x \in X\right\},$$

$$M_{j^{*}_{\alpha,\beta}}(A) = [N_{AL}(x),\ (N_{AU}(x)^{\delta} + \alpha(1 - \beta M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta}))^{\frac{1}{\delta}}],$$

$$N_{j^{*}_{\alpha,\beta}}(A) = [\beta^{\frac{1}{\delta}} M_{AL}(x),\ \beta^{\frac{1}{\delta}} M_{AU}(x)].$$

Theorem 3.2. *For every* $A \in GIVIFS_B$, *and for every three real numbers* $\alpha, \beta, \gamma \in [0, 1]$

(i) $j^{*}_{\alpha,\beta}(A) \in GIVIFS_B$,

(ii) $\alpha \leq \gamma \Rightarrow j^{*}_{\alpha,\beta}(A) \subset j^{*}_{\gamma,\beta}(A)$,

(iii) $\beta \leq \gamma \Rightarrow j^{*}_{\alpha,\beta}(A) \supset j^{*}_{\alpha,\gamma}(A)$,

(iv) $j^{*}_{1,1}(A) = \overline{\square A}$,

(v) $j^{*}_{0,1}(A) = \overline{A}$.

Proof. (i)

$$M_{j^*_{\alpha,\beta}(A)U}(x)^{\delta} + N_{j^*_{\alpha,\beta}(A)U}(x)^{\delta}$$

$$= \left(\left(N_{AU}(x)^{\delta} + \alpha(1 - \beta M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta})\right)^{\frac{1}{\delta}}\right)^{\delta}$$

$$+ \left(\beta^{\frac{1}{\delta}} M_{AU}(x)\right)^{\delta}$$

$$= \left(N_{AU}(x)^{\delta} + \alpha(1 - \beta M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta})\right) + \beta M_{AU}(x)^{\delta}$$

$$\leq N_{AU}(x)^{\delta} + 1 - \beta M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta} + \beta M_{AU}(x)^{\delta} = 1.$$

Finally, it can be concluded that $j^*_{\alpha,\beta}(A) \in GIVIFS_B$.

(ii) Since $\alpha \leq \gamma$, then it is clear that

$$\left[N_{AL}(x), \left(N_{AU}(x)^{\delta} + \alpha(1 - \beta M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta})\right)^{\frac{1}{\delta}}\right]$$

$$\leq \left[N_{AL}(x), \left(N_{AU}(x)^{\delta} + \gamma(1 - \beta M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta})\right)^{\frac{1}{\delta}}\right].$$

Finally, we have $j^*_{\alpha,\beta}(A) \subset j^*_{\gamma,\beta}(A)$.

The proof of (iii) is similar to that of (ii). Proofs (iv) and (v) are obvious.

Definition 3.3. Let $\alpha, \beta \in [0, 1]$ and $A \in GIVIFS_B$, we define the operator of $H^*_{\alpha,\beta}(A)$ as follows:

$$H^*_{\alpha,\beta}(A) = \left\{\langle x, M_{H^*_{\alpha,\beta}}(A), N_{H^*_{\alpha,\beta}}(A)\rangle : x \in X\right\},$$

$$M_{H^*_{\alpha,\beta}}(A) = [\alpha^{\frac{1}{\delta}} M_{AL}(x), \alpha^{\frac{1}{\delta}} M_{AU}(x)],$$

$$N_{H^*_{\alpha,\beta}}(A) = [N_{AL}(x), (N_{AU}(x)^{\delta} + \beta(1 - \alpha M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta}))^{\frac{1}{\delta}}].$$

Theorem 3.3. *For every* $A \in GIVIFS_B$, *and for every three real numbers* $\alpha, \beta, \gamma \in [0, 1]$,

(i) $H^*_{\alpha,\beta}(A) \in GIVIFS_B$,

(ii) $\alpha \le \gamma \Rightarrow H^*_{\alpha,\beta}(A) \subset H^*_{\gamma,\beta}(A)$,

(iii) $\beta \le \gamma \Rightarrow H^*_{\alpha,\beta}(A) \supset H^*_{\alpha,\gamma}(A)$,

(iv) $H^*_{1,0}(A) = A$,

(v) $H^*_{1,1}(A) = \Box A$.

Proof. (i)

$$M_{H^*_{\alpha,\beta}(A)U}(x)^{\delta} + N_{H^*_{\alpha,\beta}(A)U}(x)^{\delta}$$

$$= \left(\alpha^{\frac{1}{\delta}} M_{AU}(x)\right)^{\delta} + \left((N_{AU}(x)^{\delta} + \beta(1 - \alpha M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta}))^{\frac{1}{\delta}}\right)^{\delta}$$

$$= \alpha M_{AU}(x)^{\delta} + (N_{AU}(x)^{\delta} + \beta(1 - \alpha M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta}))$$

$$\le \alpha M_{AU}(x)^{\delta} + (N_{AU}(x)^{\delta} + (1 - \alpha M_{AU}(x)^{\delta} - N_{AU}(x)^{\delta})) = 1.$$

Finally, it can be concluded that $H^*_{\alpha,\beta}(A) \in GIVIFS_B$.

The proofs of (ii), (iii), (iv), and (v) are obvious.

Definition 3.4. Let $\alpha, \beta \in [0, 1]$ and $A \in GIVIFS_B$, we define the operator of $h^*_{\alpha,\beta}(A)$ as follows:

$$h^*_{\alpha,\beta}(A) = \left\{\langle x, M_{h^*_{\alpha,\beta}}(A), N_{h^*_{\alpha,\beta}}(A)\rangle : x \in X\right\},$$

$$M_{h^*_{\alpha,\beta}(A)}(A) = [\alpha^{\frac{1}{\delta}} N_{AL}(x),\ \alpha^{\frac{1}{\delta}} N_{AU}(x)],$$

$$N_{h^*_{\alpha,\beta}(A)}(A) = [(M_{AL}(x),\ (M_{AU}(x)^{\delta} + \beta(1 - M_{AU}(x)^{\delta} - \alpha N_{AU}(x)^{\delta}))^{\frac{1}{\delta}}].$$

Theorem 3.4. *For every* $A \in GIVIFS_B$, *and for every three real numbers* $\alpha, \beta, \gamma \in [0, 1]$

(i) $h^*_{\alpha,\beta}(A) \in GIVIFS_B$,

(ii) $\alpha \leq \gamma \Rightarrow h^*_{\alpha,\beta}(A) \subset h^*_{\gamma,\beta}(A)$,

(iii) $\beta \leq \gamma \Rightarrow h^*_{\alpha,\beta}(A) \supset h^*_{\alpha,\gamma}(A)$,

(iv) $h^*_{1,0}(A) = \overline{A}$,

(v) $h^*_{1,1}(A) = \overline{\Diamond A}$.

Proof. The proofs are obvious.

Corollary 3.1. *Let* $A \in GIVIFS_B$, *we have*

(i) $j^*_{\alpha,\beta}(\overline{A}) = J^*_{\alpha,\beta}(A)$,

(ii) $h^*_{\alpha,\beta}(\overline{A}) = H^*_{\alpha,\beta}(A)$,

(iii) $\overline{J^*_{\beta,\alpha}(A)} = h^*_{\alpha,\beta}(A)$,

(iv) $\overline{j^*_{\beta,\alpha}(A)} = H^*_{\alpha,\beta}(A)$.

Theorem 3.5. *For every* $GIVIFS_B s$ $A, B, A \subset B$ *and for every two real numbers* $\alpha, \beta \in [0, 1]$, *we have*

(i) $J^*_{\alpha,\beta}(A) \subset J^*_{\alpha,\beta}(B)$,

(ii) $j^*_{\alpha,\beta}(A) \supset j^*_{\alpha,\beta}(B)$,

(iii) $H^*_{\alpha,\beta}(A) \subset H^*_{\alpha,\beta}(B)$,

(iv) $h^*_{\alpha,\beta}(A) \supset h^*_{\alpha,\beta}(B)$,

(v) $H^*_{\alpha,\beta}(A) \subset A \subset J^*_{\alpha,\beta}(A)$.

Proof. The proofs are obvious.

Theorem 3.6. *For every* $GIVIFS_B$ A, *and for every two real numbers* $\alpha, \beta \in [0, 1]$, *we have*

(i) $\Diamond A \subset \Diamond J^*_{\alpha,\beta}(A)$,

(ii) $\Box A \subset \Box J^*_{\alpha,\beta}(A)$,

(iii) $\Diamond H^*_{\alpha,\beta}(A) \subset \Diamond A$,

(iv) $\Box H^*_{\alpha,\beta}(A) \subset \Box A$.

Proof. The proofs are obvious.

Remark 3.1. According to definition, the operators of $J^*_{\alpha,\beta}(A)$ increases the membership degree A and reduces non-membership degree A, the operators of $j^*_{\alpha,\beta}(A)$ increases the membership degree $\overline{A}$ and reduces non-membership degree $\overline{A}$, the operators of $H^*_{\alpha,\beta}(A)$ reduces the membership degree A and increases non-membership degree A, the operators of $h^*_{\alpha,\beta}(A)$ reduces the membership degree $\overline{A}$ and increases non-membership degree $\overline{A}$.

Example 3.1. Let $A = \{\langle x_1, [0.4, 0.5], [0.1, 0.2]\rangle\}$, $\delta = 0.5$, then

$\Box A = \{\langle x_1, [0.4, 0.5], [0.1, 0.0858]\rangle\}$, $\Diamond A = \{\langle x_1, [0.4, 0.3056], [0.1, 0.2]\rangle\}$,

$J^*_{\alpha,\beta}(A) = \{\langle x_1, [0.4, (0.7071 + \alpha(0.2929 - 0.4472\beta))^2], [0.1\beta^2, 0.2\beta^2]\rangle\}$,

$$j^{*}_{\alpha,\beta}(A) = \{\langle x_1, [0.1, (0.4472 + \alpha(0.5528 - 0.7071\beta))^2], [0.4\beta^2, 0.5\beta^2]\rangle\},$$

$$H^{*}_{\alpha,\beta}(A) = \{\langle x_1, [0.4\alpha^2, 0.5\alpha^2], [0.1, (0.4472 + \beta(0.5528 - 0.7071\alpha))^2]\rangle\},$$

$$h^{*}_{\alpha,\beta}(A) = \{\langle x_1, [0.1\beta^2, 0.2\beta^2], [0.4, (0.7071 + \beta(0.2929 - 0.4472\alpha))^2]\rangle\}.$$

4. Conclusion

We have introduced four modal types of operators over Baloui's generalized interval valued intuitionistic fuzzy sets and their relationships are proved. Some related results have been proved.

References

[1] K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1) (1986), 87-96.

[2] K. T. Atanassov and G. Gargov, Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 31(3) (1989), 343-349.

[3] K. T. Atanassov, Operators over interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 64(2) (1994), 159-174.

[4] K. T. Atanassov, Intuitionistic Fuzzy Sets, Theory and Applications, Springer Verlag, New York, 1999.

[5] E. Baloui Jamkhaneh and S. Nadarajah, A new generalized intuitionistic fuzzy sets, Hacettepe Journal of Mathematics and Statistics 44(6) (2015), 1537-1551

[6] E. Baloui Jamkhaneh, New generalized interval value intuitionistic fuzzy sets, Research and Communications in Mathematics and Mathematical Sciences 5(1) (2015), 33-46.

[7] M. Bhowmik and M. Pal, Some results on generalized interval-valued intuitionistic fuzzy sets, International Journal of Fuzzy Systems 14(2) (2012).

[8] M. Bhowmik and M. Pal, Partition of generalized interval valued intuitionistic fuzzy sets and some properties, International Journal of Applied Mathematical Analysis and Applications 4(1) (2009), 1-10.

[9] M. Bhowmik and M. Pal, Generalized interval valued intuitionistic fuzzy sets, The Journal of Fuzzy Mathematics 18(2) (2010), 357-371.

[10] S. M. Chen, M. W. Yang, S. W. Yang and C. J. Liau, Multicriteria fuzzy decision making based on interval valued intuitionistic fuzzy sets, Expert Systems with Applications 39(15) (2012), 12085-12091.

[11] D. Dubois and H. Prade, Fuzzy Sets and Systems: Theory and Applications, Academic Press, New York, 1980.

[12] W. L. Gau and D. J. Buehrer, Vague sets, IEEE Transactions on Systems Man and Cybernetics 23(2) (1993), 610-614.

[13] R. John, Type 2 fuzzy sets: An appraisal of theory and applications, International Journal Uncertainty Fuzziness and Knowledge-Based Systems 6(6) (1998), 563-576.

[14] D. F. Li, Linear programming method for MADM with interval valued intuitionistic fuzzy sets, Expert Systems with Applications 37(8) (2010), 5939-5945.

[15] D. F. Li, TOPSIS-based nonlinear-programming methodology for multi-attribute decision making with interval valued intuitionistic fuzzy sets, IEEE Transactions on Fuzzy Systems 18(2) (2010), 299-311.

[16] D. F. Li, Closeness coefficient based nonlinear programming method for interval valued intuitionistic fuzzy multiattribute decision making with incomplete preference information, Applied Soft Computing 11(4) (2011), 3402-3418.

[17] B. Liu and M. X. Luo, Multicriteria decision making based on interval valued intuitionistic fuzzy sets with a new kind of accuracy function, Quantitative Logic and Soft Computing (2016), 477-486.

[18] R. Sahin, Fuzzy multicriteria decision making method based on the improved accuracy function for interval valued intuitionistic fuzzy sets, Soft Computing (2015), 1-7.

[19] R. Srinivasan and N. Palaniappan, Some operations on intuitionistic fuzzy sets of root type, Notes on IFS 12(3) (2006), 20-29.

[20] S. Sudharsan and D. Ezhilmaran, Two new operator defined over interval valued intuitionistic fuzzy sets, International Journal of Fuzzy Logic Systems 4(4) (2014), 1-13.

[21] S. Sudharsan and D. Ezhilmaran, Some operators defined over intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets, International Journal of Mathematical Archive 6(10) (2015), 30-38.

[22] S. Sudharsan and D. Ezhilmaran, Weighted arithmetic average operator based on interval valued intuitionistic fuzzy values and their application to multi-criteria decision making for investment, Journal of Information and Optimization Sciences 37(2) (2016), 247-260.

[23] V. Torra and Y. Narukawa, On hesitant fuzzy sets and decision, The 18th IEEE International Conference on Fuzzy Systems, Jeju Island, Korea (2009), 1378-1382.

[24] W. Wang and X. Liu, Interval valued intuitionistic fuzzy hybrid weighted averaging operator based on Einstein operation and its application to decision making, Journal of Intelligent & Fuzzy Systems 25(2) (2013), 279-290.

[25] W. Wang, X. Liu and Y. Qin, Interval valued intuitionistic fuzzy aggregation operators, Journal of Systems Engineering and Electronics 23(4) (2012), 574-580.

[26] Z. S. Xu and C. H. E. N. Jian, Approach to group decision making based on interval valued intuitionistic judgment matrices, Systems Engineering-Theory & Practice 27(4) (2007), 126-133.

[27] Z. S. Xu, Methods for aggregating interval valued intuitionistic fuzzy information and their application to decision making, Control and Decision 22(2) (2007), 215-219.

[28] Z. S. Xu, A method based on distance measure for interval-valued intuitionistic fuzzy group decision making, Information Sciences 180(1) (2010), 181-190.

[29] R. R. Yager, On the theory of bags, International Journal of General Systems 13(1) (1986), 23-37.

[30] L. A. Zadeh, Fuzzy sets, Information and Control 8(3) (1965), 338-353.

[31] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-I, Information Sciences 8(3) (1975), 199-249.

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