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INTUITIONISTIC FUZZY *I*-CONVERGENT DIFFERENCE DOUBLE SEQUENCE SPACES

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Abstract

In this paper, we study the intuitionistic fuzzy *I*-convergent difference double sequence spaces ${}_2I_{\Delta}^{(\mu, v)}$ and ${}_2I_{\Delta}^{0(\mu, v)}$. Also we introduce a new concept, called as closed ball in these spaces. Benefiting from these notions, we establish a new topological space and investigate some topological properties in intuitionistic fuzzy *I*-convergent difference double sequence spaces ${}_2I_{\Delta}^{(\mu, v)}$ and ${}_2I_{\Delta}^{0(\mu, v)}$.

1. Introduction

Fuzzy set theory defined by Zadeh [1] has been applied various branches of mathematics such as in the theory of functions [2] and in the approximation theory [3]. Fuzzy topology plays an essential role in fuzzy theory. It deals with such conditions where the classical theories break down. The intuitionistic fuzzy normed space and intuitionistic fuzzy

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n-normed space which were investigated in [4]-[5] are the most contemporary improvements in fuzzy topology. Recently, the definition of I-convergence in intuitionistic fuzzy zweier I-convergent sequence spaces and intuitionistic fuzzy zweier I-convergent double sequence spaces have been studied in [10]-[13].

The notion of statistical convergence was introduced by Steinhaus [14] and Fast [15] has been applied for the convergence problems of matrices (double sequences) through the concept of the natural density. Some statistical convergence types in intuitionistic fuzzy normed spaces and intuitionistic fuzzy n-normed spaces were investigated in [6]-[9]. As an extended definition of statistical convergence, definition of I-convergence was introduced by Kostyrko et al. [16] by using the idea of I of subsets of the set of natural numbers. Recently, the notion of statistical convergence of double sequences $x = (x_{ij})$ has been defined and investigated in [25] and [26]. Quite recently, I and I^*-convergence of double sequences have been studied by Das et al. [17].

Some new sequence spaces were introduced by means of various matrix transformations in [21]-[23]. Kızmaz [20] defined the difference sequence spaces with the difference matrix as follows:

$$X(\Delta) = \{x = (x_k) : \Delta x \in X\},$$

for $X = l_\infty, c, c_0$, where $\Delta x_k = x_k - x_{k+1}$ and Δ denotes the difference matrix $\Delta = (\Delta_{nk})$ defined by

$$\Delta_{nk} = \begin{cases} (-1)^{n-k}, \text{ if } n \leq k \leq n+1, \\ \quad 0, \text{ if } 0 \leq k < n. \end{cases}$$

In this study, we introduce the intuitionistic fuzzy I-convergent difference double sequence spaces ${}_2I_\Delta^{(\mu, v)}$ and ${}_2I_\Delta^{0(\mu, v)}$ and investigate some topological properties of these new spaces.

2. Basic Definitions

In this section, we give some definitions and notations which will be used for this investigation.

Definition 2.1 ([18]). A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is said to be a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative,

(ii) $*$ is continuous,

(iii) $a * 1 = a$ for all $a \in [0, 1]$,

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0, 1]$.

Definition 2.2 ([18]). A binary operation $\circ : [0, 1] \times [0, 1] \to [0, 1]$ is said to be a continuous t-conorm if it satisfies the following conditions:

(i) $\circ$ is associative and commutative,

(ii) $\circ$ is continuous,

(iii) $a \circ 0 = a$ for all $a \in [0, 1]$,

(iv) $a \circ b \leq c \circ d$ whenever $a \leq c$ and $b \leq d$ for each $a, b, c, d \in [0, 1]$.

Definition 2.3 ([4]). The five-tuple $(X, \mu, \upsilon, *, \circ)$ is said to be intuitionistic fuzzy normed linear space (or shortly IFNLS) where X is a linear space over a field F, $*$ is a continuous t-norm, $\circ$ is a continuous t-conorm, μ, υ are fuzzy sets on $X \times (0, \infty)$, μ denotes the degree of membership and υ denotes the degree of nonmembership of $(x, t) \in X \times (0, \infty)$ satisfying the following conditions for every $x, y \in X$ and $s, t > 0$:

(i) $\mu(x, t) + \upsilon(x, t) \leq 1$,

(ii) $\mu(x, t) > 0$,

(iii) $\mu(x, t) = 1$ if and only if $x = 0$,

(iv) $\mu(\alpha x, t) = \mu\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \neq 0$,

(v) $\mu(x, t) * \mu(y, s) \leq \mu(x + y, t + s)$,

(vi) $\mu(x, .) : (0, \infty) \to [0, 1]$ is continuous,

(vii) $\lim_{t \to \infty} \mu(x, t) = 1$ and $\lim_{t \to 0} \mu(x, t) = 0$,

(viii) $v(x, t) < 1$,

(ix) $v(x, t) = 0$ if and only if $x = 0$,

(x) $v(\alpha x, t) = v\left(x, \frac{t}{|\alpha|}\right)$ if $\alpha \neq 0$,

(xi) $v(x, t) \circ v(y, s) \geq v(x + y, s + t)$,

(xii) $v(x, .) : (0, \infty) \to [0, 1]$ is continuous,

(xiii) $\lim_{t \to \infty} v(x, t) = 0$ and $\lim_{t \to 0} v(x, t) = 1$.

In this case (μ, v) is called intuitionistic fuzzy linear norm.

Example 2.1 ([4]). Let $(X, \|.\|)$ be a normed linear space, and let $a * b = ab$ and $a \circ b = \min\{a + b, 1\}$ for all $a, b \in [0, 1]$. For all $x \in X$ and every $t > 0$, consider

$$\mu(x, t) := \frac{t}{t + \|x\|} \text{ and } v(x, t) := \frac{\|x\|}{t + \|x\|}.$$

Then $(X, \mu, v, *, \circ)$ is an IFNLS.

Definition 2.4 ([4]). Let $(X, \mu, v, *, \circ)$ be an IFNLS. A sequence $x = (x_k)$ in X is convergent to $L \in X$ with respect to the intuitionistic fuzzy linear norm (μ, v) if, for every $\varepsilon > 0$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $\mu(x_k - L, t) > 1 - \varepsilon$ and $v(x_k - L, t) < \varepsilon$ for all $k \geq k_0$ where $k \in \mathbb{N}$. It is denoted by $(\mu, v) - \lim x = L$.

Theorem 2.1 ([19]). *Let* $(X, \mu, v, *, \circ)$ *be an IFNLS. Then, a sequence* $x = (x_k)$ *in X is convergent to* $L \in X$ *if and only if* $\lim_{k \to \infty} \mu(x_k - L, t) = 1$ *and* $\lim_{k \to \infty} v(x_k - L, t) = 0.$

Definition 2.5 ([16]). If X is a non-empty set, then a family of sets $I \subset P(X)$ is called an ideal in X if and only if

(i) $\emptyset \in I,$

(ii) for each $A, B \in I$ implies that $A \cup B \in I,$ and

(iii) for each $A \in I$ and $B \subset A$ we have $B \in I,$

where $P(X)$ is the power set of X.

Definition 2.6 ([16]). If X is a non-empty set, then a non-empty family of sets $F \subset P(X)$ is called a filter on X if and only if

(i) $\emptyset \notin F,$

(ii) for each $A, B \in F$ implies that $A \cap B \in F,$ and

(iii) for each $A \in F$ and $A \supset B,$ we have $B \in F.$

An ideal I is called non-trivial if $I \neq \emptyset$ and $X \notin I.$ A non-trivial ideal $I \subset P(X)$ is called an admissible ideal in X if and only if $\{\{x\} : x \in X\} \subseteq I.$

A relation between the concepts of an ideal and a filter is given by the following proposition.

Proposition 2.1 ([16]). *Let* $I \subset P(X)$ *be a non-trivial ideal. Then the class* $F = F(I) = \{M \subset X : M = X - A, \textit{ for some } A \in I\}$ *is a filter on* $X.F = F(I)$ *is called the filter associated with the ideal I.*

Definition 2.8 ([24]). Let ${}_2I$ be a non-trivial ideal of $\mathbb{N} \times \mathbb{N}$ and $(X, \mu, v, *, \circ)$ be an IFNLS. A double sequence $x = (x_{ij})$ of elements of X is said to be ${}_2I$-convergent to $L \in X$ with respect to the intuitionistic fuzzy linear norm (μ, v) if, for every $\varepsilon > 0$ and $t > 0$, the set

$$\{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(x_{ij} - L, t) \leq 1 - \varepsilon \text{ or } v(x_{ij} - L, t) \geq \varepsilon\} \in I.$$

In this case, we write $I_2^{(\mu, v)} - \lim x = L$.

3. Main Results

In this study, we defined a variant of ideal convergent sequence spaces called intuitionistic fuzzy ideal difference convergent double sequence spaces and investigated some topological properties of these spaces.

Let ${}_2w$ be the space of all real double sequences. Intuitionistic fuzzy I-convergent difference double sequence spaces are defined as:

$${}_2I_{\Delta}^{(\mu, v)} = \{(x_{ij}) \in {}_2w : \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta x_{ij} - L, t) \leq 1 - \varepsilon \text{ or } v(\Delta x_{ij} - L, t) \geq \varepsilon\} \in I_2\},$$

and

$${}_2I_{\Delta}^{0(\mu, v)} = \{(x_{ij}) \in {}_2w : \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta x_{ij}, t) \leq 1 - \varepsilon \text{ or } v(\Delta x_{ij}, t) \geq \varepsilon\} \in I_2\}.$$

Moreover, an open ball ${}_2B_x(r, t)$ with centre $x \in {}_2I_{\Delta}^{(\mu, v)}$ and radius $r \in (0, 1)$ with respect to t, is defined as follows:

$${}_2B_x(r, t) = \{y \in {}_2I_{\Delta}^{(\mu, v)} : \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta x_{ij} - \Delta y_{ij}, t) \leq 1 - r \text{ or } v(\Delta x_{ij} - \Delta y_{ij}, t) \geq r\} \in I_2\}.$$

Theorem 3.1. ${}_2I_{\Delta}^{(\mu, v)}$ *and* ${}_2I_{\Delta}^{0(\mu, v)}$ *are linear spaces.*

Proof. We prove the result for ${}_2I_{\Delta}^{(\mu, v)}$. Similarly, it can be proved for ${}_2I_{\Delta}^{0(\mu, v)}$. Let $(x_{ij}), (y_{ij}) \in {}_2I_{\Delta}^{(\mu, v)}$ and α, β be scalars. The proof is trivial for $\alpha = 0$ and $\beta = 0$. Let $\alpha \neq 0$ and $\beta \neq 0$. For a given $\varepsilon > 0$, choose $s > 0$ such that $(1 - \varepsilon) * (1 - \varepsilon) > 1 - s$ and $\varepsilon \circ \varepsilon < s$. Hence, we have

$$A_1 = \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta x_{ij} - L_1, t/2|\alpha|) \leq 1 - \varepsilon$$

$$\text{or } v(\Delta x_{ij} - L_1, t/2|\alpha|) \geq \varepsilon\} \in I_2,$$

$$A_2 = \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta y_{ij} - L_2, t/2|\beta|) \leq 1 - \varepsilon$$

$$\text{or } v(\Delta x_{ij} - L_1, t/2|\beta|) \geq \varepsilon\} \in I_2,$$

$$A_1^c = \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta x_{ij} - L_1, t/2|\alpha|) > 1 - \varepsilon \text{ and } v(\Delta x_{ij} - L_1,$$

$$t/2|\alpha|) < \varepsilon\} \in F(I_2),$$

and

$$A_2^c = \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta y_{ij} - L_2, t/2|\beta|) > 1 - \varepsilon \text{ and } v(\Delta y_{ij} - L_2,$$

$$t/2|\beta|) < \varepsilon\} \in F(I_2).$$

Let define the set $A_3 = A_1 \cup A_2$. Hence $A_3 \in I_2$. It follows that A_3^c is a nonempty set in $F(I_2)$. We will prove that for every $(x_{ij}), (y_{ij}) \in {}_2I_{\Delta}^{(\mu, v)}$,

$$A_3^c \subset \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu((\alpha.\Delta x_{ij} + \beta.\Delta y_{ij}) - (\alpha L_1 + \beta L_2), t) > 1 - s$$

$$\text{and } v((\alpha.\Delta x_{ij} + \beta.\Delta y_{ij}) - (\alpha L_1 + \beta L_2), t) < s\}.$$

Let $(m, n) \in A_3^c$. Then

$$\mu(\Delta x_{mn} - L_1, t/2|\alpha|) > 1 - \varepsilon \quad \text{and} \quad \upsilon(\Delta x_{mn} - L_1, t/2|\alpha|) < \varepsilon,$$

and

$$\mu(\Delta y_{mn} - L_2, t/2|\beta|) > 1 - \varepsilon \quad \text{and} \quad \upsilon(\Delta y_{mn} - L_2, t/2|\beta|) < \varepsilon.$$

In this case,

$$\mu((\alpha.\Delta x_{mn} + \beta.\Delta y_{mn}) - (\alpha L_1 + \beta L_2), t) \geq \mu(\alpha.\Delta x_{mn} - \alpha L_1, t/2) * \mu(\beta.\Delta y_{mn} - \beta L_2, t/2)$$

$$= \mu(\Delta x_{mn} - L_1, t/2|\alpha|) * \mu(\Delta y_{mn} - L_2, t/2|\beta|) > (1 - \varepsilon) * (1 - \varepsilon) > 1 - s,$$

and

$$\upsilon((\alpha.\Delta x_{mn} + \beta.\Delta y_{mn}) - (\alpha L_1 + \beta L_2), t) \leq \upsilon(\alpha.\Delta x_{mn} - \alpha L_1, t/2) \circ \upsilon(\beta.\Delta y_{mn} - \beta L_2, t/2)$$

$$= \upsilon(\Delta x_{mn} - L_1, t/2|\alpha|) \circ \upsilon(\Delta y_{mn} - L_2, t/2|\beta|) < \varepsilon \circ \varepsilon < s.$$

This proves that

$$A_3^c \subset \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu((\alpha.\Delta x_{ij} + \beta.\Delta y_{ij}) - (\alpha L_1 + \beta L_2), t) > 1 - s$$
$$\text{and } \upsilon((\alpha.\Delta x_{ij} + \beta.\Delta y_{ij}) - (\alpha L_1 + \beta L_2), t) < s\}.$$

Hence ${}_2I_\Delta^{(\mu, \upsilon)}$ is a linear space.

Theorem 3.2. *Every closed ball* ${}_2B_x^c(r, t)$ *is an open set in* ${}_2I_\Delta^{(\mu, \upsilon)}$.

Proof. Let ${}_2B_x(r, t)$ be an open ball with centre $x \in {}_2I_\Delta^{(\mu, \upsilon)}$ and radius $r \in (0, 1)$ with respect to t, i.e.,

$${}_2B_x(r, t) = \{y \in {}_2I_\Delta^{(\mu, \upsilon)} : \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta x_{ij} - \Delta y_{ij}, t) \leq 1 - r \text{ or } \upsilon(\Delta x_{ij} - \Delta y_{ij}, t) \geq r\} \in I_2\}.$$

Let $y \in {}_2B_x^c(r, t)$. Then $\mu(\Delta x - \Delta y, t) > 1 - r$ and $v(\Delta x - \Delta y, t) < r$.

Since $\mu(\Delta x - \Delta y, t) > 1 - r$, there exists $t_0 \in (0, t)$ such that $\mu(\Delta x - \Delta y, t_0) > 1 - r$ and $v(\Delta x - \Delta y, t_0) < r$.

Let $r_0 = \mu(\Delta x - \Delta y, t_0)$. Since $r_0 > 1 - r$, there exists $s \in (0, 1)$ such that $r_0 > 1 - s > 1 - r$ and so there exists $r_1, r_2 \in (0, 1)$ such that $r_0 * r_1 > 1 - s$ and $(1 - r_0) \circ (1 - r_2) < s$.

Let $r_3 = \max\{r_1, r_2\}$. Then $1 - s < r_0 * r_1 \leq r_0 * r_3$ and $(1 - r_0) \circ (1 - r_3) \leq (1 - r_0) \circ (1 - r_2) < s$.

Consider the closed balls ${}_2B_y^c(1 - r_3, t - t_0)$ and ${}_2B_x^c(r, t)$. We prove that ${}_2B_y^c(1 - r_3, t - t_0) \subset {}_2B_x^c(r, t)$. Let $z \in {}_2B_y^c(1 - r_3, t - t_0)$. Then $\mu(\Delta y - \Delta z, t - t_0) > r_3$ and $v(\Delta y - \Delta z, t - t_0) < 1 - r_3$. Hence

$$\mu(\Delta x - \Delta z, t) \geq \mu(\Delta x - \Delta y, t_0) * \mu(\Delta y - \Delta z, t - t_0) > r_0 * r_3 \geq r_0 * r_1 > 1 - s > 1 - r,$$

and

$$v(\Delta x - \Delta z, t) \leq v(\Delta x - \Delta y, t_0) \circ v(\Delta y - \Delta z, t - t_0) < (1 - r_0) \circ (1 - r_3) < s < r.$$

Thus $z \in {}_2B_x^c(r, t)$ and hence ${}_2B_y^c(1 - r_3, t - t_0) \subset {}_2B_x^c(r, t)$.

Remark 3.1. It is clear that ${}_2I_\Delta^{(\mu, v)}$ is an IFNLS. Define

$${}_2\tau_\Delta^{(\mu, v)} = \Big\{A \subset {}_2I_\Delta^{(\mu, v)} : \text{for each } x \in A \text{ there exist } t > 0 \text{ and } r \in (0, 1) \text{ such that } {}_2B_x^c(r, t) \subset A\Big\}.$$

Then ${}_2\tau_\Delta^{(\mu, v)}$ is a topology on ${}_2I_\Delta^{(\mu, v)}$.

Theorem 3.3. *The topology* ${}_2\tau_\Delta^{(\mu, v)}$ *on* ${}_2I_\Delta^{0(\mu, v)}$ *is first countable.*

Proof. It is clear that $\{{}_2B_x^c(\frac{1}{n}, \frac{1}{n}) : n \in \mathbb{N}\}$ is a local base at $x \in {}_2I_\Delta^{(\mu, v)}$. Then the topology ${}_2\tau_\Delta^{(\mu, v)}$ on ${}_2I_\Delta^{0(\mu, v)}$ is first countable.

Theorem 3.4. *${}_2I_\Delta^{(\mu, v)}$ and ${}_2I_\Delta^{0(\mu, v)}$ are Hausdorff spaces.*

Proof. Let $x, y \in {}_2I_\Delta^{(\mu, v)}$ such that $x \neq y$. Then $0 < \mu(\Delta x - \Delta y, t) < 1$ and $0 < v(\Delta x - \Delta y, t) < 1$.

Let define r_1, r_2 and r such that $r_1 = \mu(\Delta x - \Delta y, t)$, $r_2 = v(\Delta x - \Delta y, t)$, and $r = \max\{r_1, 1 - r_2\}$. Then for each $r_0 \in (r, 1)$ there exist r_3 and r_4 such that $r_3 * r_4 \geq r_0$ and $(1 - r_3) \circ (1 - r_4) \leq (1 - r_0)$.

Let $r_5 = \max\{r_3, (1 - r_4)\}$ and consider the closed balls ${}_2B_x^c(1 - r_5, \frac{t}{2})$ and ${}_2B_y^c(1 - r_5, \frac{t}{2})$. Then clearly ${}_2B_x^c(1 - r_5, \frac{t}{2}) \cap {}_2B_y^c(1 - r_5, \frac{t}{2}) = \emptyset$.

Suppose that $z \in {}_2B_x^c(1 - r_5, \frac{t}{2})_2 \cap B_y^c(1 - r_5, \frac{t}{2})$, then

$$r_1 = \mu(\Delta x - \Delta y, t) \geq \mu(\Delta x - \Delta z, \frac{t}{2}) * \mu(\Delta y - \Delta z, \frac{t}{2}) \geq r_5 * r_5 \geq r_3 * r_4 \geq r_0 > r,$$

and

$$r_2 = v(\Delta x - \Delta y, t) \leq v(\Delta x - \Delta z, \frac{t}{2}) \circ v(\Delta y - \Delta z, \frac{t}{2}) \leq (1 - r_5) \circ (1 - r_5)$$

$$\leq (1 - r_3) \circ (1 - r_4) \leq (1 - r_0) < 1 - r,$$

which is a contradiction. Hence ${}_2I_\Delta^{(\mu, v)}$ is a Hausdorff space.

Theorem 3.5. *Let ${}_2I_\Delta^{(\mu, v)}$ be an IFNLS, ${}_2\tau_\Delta^{(\mu, v)}$ be a topology on ${}_2I_\Delta^{(\mu, v)}$ and (x_{ij}) be a sequence in ${}_2I_\Delta^{(\mu, v)}$. Then a sequence (x_{ij}) is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ, v) if and only if $\mu(\Delta x_{ij} - \Delta x_0, t) \to 1$ and $v(\Delta x_{ij} - \Delta x_0, t) \to 0$ as $i \to \infty$, $j \to \infty$.*

Proof. Let ${}_2B_{x_0}(r, t)$ be an open ball with centre $x_0 \in {}_2I_\Delta{}^{(\mu, v)}$ and radius $r \in (0, 1)$ with respect to t, i.e.,

$$ {}_2B_{x_0}(r, t) = \{(x_{ij}) \in {}_2I_\Delta{}^{(\mu, v)} : \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta x_{ij} - \Delta x_0, t) \leq 1 - r \text{ or } v(\Delta x_{ij} - \Delta x_0, t) \geq r\} \in I_2\}. $$

Suppose that a sequence (x_{ij}) is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ, v). Then for $r \in (0, 1)$ and $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $(x_{ij}) \in B^c_{x_0}(r, t)$ for all $i \geq k_0$, $j \geq k_0$. Thus

$$ \{(i, j) \in \mathbb{N} \times \mathbb{N} : \mu(\Delta x_{ij} - \Delta x_0, t) > 1 - r \text{ and } v(\Delta x_{ij} - \Delta x_0, t) < r\} \in F(I_2). $$

So $1 - \mu(\Delta x_{ij} - \Delta x_0, t) < r$ and $v(\Delta x_{ij} - \Delta x_0, t) < r$, for all $i \geq k_0$, $j \geq k_0$. Then $\mu(\Delta x_{ij} - \Delta x_0, t) \to 1$ and $v(\Delta x_{ij} - \Delta x_0, t) \to 0$ as $i \to \infty$, $j \to \infty$.

Conversely, suppose that for each $t > 0$, $\mu(\Delta x_{ij} - \Delta x_0, t) \to 1$ and $v(\Delta x_{ij} - \Delta x_0, t) \to 0$ as $i \to \infty$, $j \to \infty$. Then, for $r \in (0, 1)$, there exists $k_0 \in \mathbb{N}$ such that $1 - \mu(\Delta x_{ij} - \Delta x_0, t) < r$ and $v(\Delta x_{ij} - \Delta x_0, t) < r$ for all $i \geq k_0$, $j \geq k_0$. So, $\mu(\Delta x_{ij} - \Delta x_0, t) > 1 - r$ and $v(\Delta x_{ij} - \Delta x_0, t) < r$ for all $i \geq k_0$, $j \geq k_0$. Hence $(x_{ij}) \in {}_2B^c_{x_0}(r, t)$ for all $i \geq k_0$, $j \geq k_0$. This proves that a sequence (x_{ij}) is Δ-convergent to Δx_0 with respect to the intuitionistic fuzzy linear norm (μ, v).

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