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A VALUE AND AMBIGUITY-BASED RANKING METHOD OF GENERALIZED INTUITIONISTIC FUZZY NUMBERS

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Abstract

In this paper, we study the concepts of values and ambiguities of the degree of membership and the degree of non-membership for generalized intuitionistic fuzzy numbers $(\mathrm{GIFN_B}s)$ due to Shabani and Baloui Jamkhaneh [12]. Also, this paper focuses on the study of value index and ambiguity index of $\mathrm{GIFN_B}$ and based on these two indices, we develop an algorithm for ranking of $\mathrm{GIFN_B}$s.

1. Introduction

Intuitionsitic fuzzy numbers and ranking them play a vital role in decision making, linear programming, transportation problem, and other intuitionistic fuzzy applications. Various definitions of intuitionistic fuzzy

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numbers and ranking methods have been proposed in literature research. Chen and Hwang [4] introduced a ranking method based on scorings of intuitionistic fuzzy numbers. Mitchell [10] introduced a ranking method for intuitionistic fuzzy number considering intuitionistic fuzzy numbers as an ensemble of fuzzy numbers. Mahapatra and Roy [9] presented triangular intuitionistic fuzzy number and used it for reliability evaluation. Wang [13] gave the definition of trapezoidal intuitionistic fuzzy numbers (TrIFNs) and interval intuitionistic fuzzy numbers. Further Wang and Zhang [14] defined the trapezoidal intuitionistic fuzzy numbers and gave a ranking method which transformed the ranking of TrIFN in to ranking of interval numbers. Nehi [11] generalized the concept of value and ambiguity for the membership and non-membership functions. Li [8] developed a ratio ranking method for triangular intuitionistic fuzzy numbers and applied to multi-attribute decision making. Dubey and Mehra [6] presents an approach based on value and ambiguity indices defined in (Li [8]) to solve linear programming problems with data as triangular intuitionistic fuzzy numbers. Zeng et al. [15] developed a value and ambiguity-based ranking method and applied to solve multi-attribute decision making problems in which the ratings of alternatives on attributes are expressed by using TrIFNs. Das and De [5] studied of two characteristics of TrIFNs, viz., value index and ambiguity index. Based on these two indices, they develop an algorithm for ranking of TrIFNs. Beaula and Priyadharsini [3] considered fuzzy transportation problem with the value and ambiguity indices of TrIFNs. Then, the stepping stone method is adopted to solve the reduced intuitionistic fuzzy transportation problem to obtain the optimal solution. Keikha and Nehi [7] considering operations and ranking methods for intuitionistic fuzzy numbers.

Baloui Jamkhaneh and Nadarajah [2] considered a new generalized intuitionistic fuzzy sets $(\mathrm{GIFS_B})$ and introduced some operators over $\mathrm{GIFS_B}$. Shabani and Baloui Jamkhaneh [12] introduced a new generalized intuitionistic fuzzy number $\mathrm{GIFN_B}$ based on generalization of the IFS related to Baloui Jamkhaneh and Nadarajah [2]. The main objective of this paper is to introduced value index and ambiguity index GIFN_B and develop an algorithm for ranking of GIFN_Bs. The originality of this study comes from the fact that, there was no previous work introduce value index and ambiguity index and ranking function for GIFN_Bs.

2. Generalized Intuitionistic Fuzzy Numbers

We collect some basic definitions and notations related to $GIFN_B(X)$.

Definition 2.1 (Baloui Jamkhaneh and Nadarajah [2]). Let X be a non-empty set. A new generalized intuitionistic fuzzy sets $(GIFS_B(X))$ A in X, is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$, where the functions $\mu_A : X \to [0, 1]$ and $\nu_A : X \to [0, 1]$, denote the degree of membership and degree of non-membership functions of A, respectively, and $0 \leq \mu_A(x)^\delta + \upsilon_A(x)^\delta \leq 1$ for each $x \in X$ and $\delta = n$ or $\frac{1}{n}$, $n = 1, 2, \ldots, N$.

Definition 2.2 (Shabani & Baloui Jamkhaneh [12]). A class of new generalized L-R type intuitionistic fuzzy number (GIFN_B) A defined as

$$\mu_A(x) = \begin{cases} \left(\frac{(x-a)\mu}{b-a}\right)^{\frac{1}{\delta}}, & a \leq x \leq b \\ \mu^{\frac{1}{\delta}}, & b \leq x \leq c \\ \left(\frac{(d-x)\mu}{d-c}\right)^{\frac{1}{\delta}}, & c \leq x \leq d \\ 0, & \text{otherwise} \end{cases}, \quad \nu_A(x) = \begin{cases} \left(\frac{(b-x)+\nu(x-a_1)}{b-a_1}\right)^{\frac{1}{\delta}}, & a_1 \leq x \leq b \\ \nu^{\frac{1}{\delta}}, & b \leq x \leq c \\ \left(\frac{(x-c)+\nu(d_1-x)}{d_1-c}\right)^{\frac{1}{\delta}}, & c \leq x \leq d_1 \\ 1, & \text{otherwise} \end{cases},$$

$\mu_A(x)$ and $\nu_A(x)$ are the functions of the membership function and the non-membership function, respectively, $a_1 \leq a \leq b \leq c \leq d \leq d_1$ and $0 \leq \mu_A(x)^\delta + \nu_A(x)^\delta \leq 1$, $\forall x \in X$.

Remark 2.1. A $\mathrm{GIFN_B}$ is said to be symmetric $\mathrm{GIFN_B}$ if $b - a = d - c$ and $b - a_1 = d_1 - c$.

Definition 2.3 (Shabani & Baloui Jamkhaneh [12]). Let $\alpha, \beta \in [0, 1]$ be fixed numbers such that $0 \le \alpha \le \mu^{\frac{1}{\delta}}, \nu^{\frac{1}{\delta}} \le \beta \le 1,\ 0 \le \alpha^{\delta} + \beta^{\delta} \le 1.$

A set of (α, β)-cut generated by a $\mathrm{GIFN_B}$ A is defined by

$$A[\alpha, \beta, \delta] = \{\langle x, \mu_A(x) \ge \alpha, \nu_A(x) \le \beta \rangle : x \in X\},$$

$A[\alpha, \beta, \delta]$ is defined as the crisp set of elements x which belong to A at least to the degree α and which does not belong to A at most to the degree β. A α-cut set of a $\mathrm{GIFN_B}$ A is a crisp subset of $\mathbb{R}$, which defined is as

$$A[\alpha, \delta] = \{\langle x, \mu_A(x) \ge \alpha, \rangle : x \in X\}, \quad 0 \le \alpha \le \mu^{\frac{1}{\delta}}.$$

According to the definition of $\mathrm{GIFN_B}$, it can be easily shown that

$$A[a, \delta] = [L_1(\alpha), U_1(\alpha)], \quad 0 \le \alpha \le \mu^{\frac{1}{\delta}},$$

$$L_1(\alpha) = a + \frac{(b-a)\alpha^{\delta}}{\mu}, \quad U_1(\alpha) = d - \frac{(d-c)\alpha^{\delta}}{\mu}.$$

Similarily a β-cut set of a $\mathrm{GIFN_B}$ A is a crisp subset of $\mathbb{R}$, which defined is as

$$A[\beta, \delta] = \{\langle x, \nu_A(x) \le \beta \rangle : x \in X\}, \quad \nu^{\frac{1}{\delta}} \le \beta \le 1.$$

According to the definition of $\mathrm{GIFN_B}$, it can be easily shown that

$$A[\beta, \delta] = [L_2(\beta), U_2(\beta)], \quad \nu^{\frac{1}{\delta}} \le \beta \le 1,$$

$$L_2(\beta) = \frac{b(1-\beta^{\delta}) + a_1(\beta^{\delta} - \nu)}{1 - v}, \quad U_2(\beta) = \frac{c(1-\beta^{\delta}) + d_1(\beta^{\delta} - \nu)}{1 - v}.$$

Remark 2.2. In special case $\mu = 1, \nu = 0,$ we have

$$L_1(\alpha) = a + (b - a)\alpha^{\delta}, \quad U_1(\alpha) = d - (d - c)\alpha^{\delta},$$

$$L_2(\beta) = b(1 - \beta^{\delta}) + a_1\beta^{\delta}, \quad U_2(\beta) = c(1 - \beta^{\delta}) + d_1\beta^{\delta}.$$

Therefore, the (α, β)-cut of a $\mathrm{GIFN_B}$ is given by

$$A[\alpha, \beta, \delta] = \{x, x \in [L_1(\alpha), U_1(\alpha)] \cap [L_2(\beta), U_2(\beta)]\} = [L(\alpha, \beta), U(\alpha, \beta)].$$

Definition 2.4 (Shabani & Baloui Jamkhaneh [12]). Let $A = (a_1', a_1, b_1, c_1, d_1, d_1', \mu_1, \nu_1, \delta)$ and $B = (a_2', a_2, b_2, c_2, d_2, d_2', \mu_2, \nu_2, \delta)$ be two $\mathrm{GIFN_B}$s; then

$$A + B = (a_1' + a_2', a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2, d_1' + d_2', \mu_1^{\delta} + \mu_2^{\delta} - \mu_1^{\delta}\mu_2^{\delta}, \nu_1^{\delta}\nu_2^{\delta}, \delta),$$

$$kA = (ka_1', ka_1, kb_1, kc_1, kd_1, kd_1', 1 - (1 - \mu_1^{\delta})^{k}, \nu_1^{k\delta}, \delta), \quad k = 2, 3, \ldots,$$

$$kA = (kd_1', kd_1, kc_1, kb_1, ka_1, ka_1', 1 - (1 - \mu_1^{\delta})^{|k|}, \nu_1^{|k|s}, \delta), \quad k = -2, -3, \ldots,$$

$$-A = (-d_1', -d_1, -c_1 - b_1, -a_1, -a_1', \mu_1, \nu_1, \delta),$$

$$A - B = (a_1' - d_2', a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2, d_1' - a_2', \mu_1^{\delta} + \mu_2^{\delta} - \mu_1^{\delta}\mu_2^{\delta}, \nu_1^{\delta}\nu_2^{\delta}, \delta).$$

3. Indices of a $\mathrm{GIFN_B}$

Definition 3.1. The (α, β)-cut of a $\mathrm{GIFN_B}$ is given by $A[\alpha, \beta] = \{x, x \in [L_1(\alpha), U_1(\alpha)] \cap [L_2(\beta), U_2(\beta)]\}$. Then the values of the membership function A and the non-membership function A are defined as follows:

$$V_{\mu}(A, \delta) = \frac{1}{2}\int_0^{\mu^{\frac{1}{\delta}}} (L_1(\alpha) + U_1(\alpha))f(\alpha)d\alpha, \quad f(\alpha) = \frac{2\alpha}{\mu^{\frac{1}{\delta}}},$$

$$V_\nu(A,\ \delta) = \frac{1}{2}\int_{\nu^{\frac{1}{\delta}}}^{1} (L_2(\beta) + U_2(\beta))f(\beta)d\beta, \quad f(\beta) = \frac{2(1-\beta)}{1-\nu^{\frac{1}{\delta}}}.$$

In special case $\mu = 1,\ \nu = 0,$ we have

$$V_\mu(A,\ \delta) = \frac{a}{2} + \frac{d}{2} + \frac{(b-a)-(d-c)}{\delta+2}, \quad V_\nu(A,\ \delta) = \frac{b}{2} + \frac{c}{2} + \frac{a_1 - b - c + d_1}{(\delta+1)(\delta+2)}.$$

In this case $V_\mu(-A,\ \delta) = -V_\mu(A,\ \delta)$ and $V_\nu(-A,\ \delta) = -V_\nu(A,\ \delta).$

Theorem 3.1. *Let* $A = (a_1', a_1, b_1, c_1, d_1, d_1', 1, 0, \delta)$ *and* $B = (a_2', a_2, b_2, c_2, d_2, d_2', 1, 0, \delta)$ *be two* $GIFN_B s$; *then*

(i) $V_\mu(A + B) = V_\mu(A) + V_\mu(B),$

(ii) $V_\mu(A - B) = V_\mu(A) + V_\mu(-B),$

(iii) $V_\nu(A + B) = V_\nu(A) + V_\nu(B),$

(iv) $V_\nu(A - B) = V_\nu(A) + V_\nu(-B),$

(v) $V_\mu(kA) = kV_\mu(A), \quad k \in \mathbb{R}^+,$

(vi) $V_\mu(kA) = |k|V_\mu(-A), \quad k \in \mathbb{R}^-,$

(vii) $V_\nu(kA) = kV_\nu(A), \quad k \in \mathbb{R}^+,$

(viii) $V_\nu(kA) = |k|V_\nu(-A), \quad k \in \mathbb{R}^-.$

Proof. See Shabani and Baloui Jamkhaneh [12].

Definition 3.2. The $(\alpha,\ \beta)$-cut of a $\mathrm{GIFN_B}$ is given by $A[\alpha,\ \beta] = \{x,\ x \in [L_1(\alpha),\ U_1(\alpha)] \cap [L_2(\beta),\ U_2(\beta)]\}$. Then the ambiguity of the membership function A and the non-membership function A are defined as follows:

$$G_\mu(A,\ \delta) = \int_0^{\mu^{\frac{1}{\delta}}} (U_1(\alpha) - L_1(\alpha))f(\alpha)d\alpha, \quad f(\alpha) = \frac{2\alpha}{\mu^{\frac{1}{\delta}}},$$

$$G_{\nu}(A, \delta) = \int_{\nu^{\frac{1}{\delta}}}^{1} (U_2(\beta) - L_2(\beta)) f(\beta) d\beta, \quad f(\beta) = \frac{2(1-\beta)}{1-\nu^{\frac{1}{\delta}}}.$$

In special case $\mu = 1, \nu = 0,$ we have

$$G_{\mu}(A, \delta) = d - a - \frac{2(b-a) + 2(d-c)}{\delta + 2}, \quad G_{\nu}(A, \delta) = c - b - \frac{2(a_1 - b) + 2(c - d_1)}{(\delta+1)(\delta+2)}.$$

It can be easily shown that $G_{\mu}(A, \delta) = G_{\mu}(-A, \delta)$, $G_{\nu}(A, \delta) = G_{\nu}(-A, \delta)$, $G_{\mu}(A, \delta) \geq 0$ and $G_{\nu}(A, \delta) \geq 0$.

Remark 3.1. In special case $\mu = 1, \nu = 0, G_{\mu}(A, \delta)$ is increasing with respect to δ, that is, $\delta_1 < \delta_2$, we have $G_{\mu}(A, \delta_1) < G_{\mu}(A, \delta_2)$.

Ramark 3.2. In special case $\mu = 1, \nu = 0, G_{\nu}(A, \delta)$ is decreasing with respect to δ, that is, $\delta_1 < \delta_2$, we have $G_{\nu}(A, \delta_1) > G_{\nu}(A, \delta_2)$.

Theorem 3.2. *Let* $A = (a_1', a_1, b_1, c_1, d_1, d_1', 1, 0, \delta)$ *and* $B = (a_2', a_2, b_2, c_2, d_2, d_2', 1, 0, \delta)$ *be two* $GIFN_B s$; *then*

(i) $G_{\mu}(A + B) = G_{\mu}(A) + G_{\mu}(B)$,

(ii) $G_{\mu}(A - B) = G_{\mu}(A) + G_{\mu}(B)$,

(iii) $G_{\nu}(A + B) = G_{\nu}(A) + G_{\nu}(B)$,

(iv) $G_{\nu}(A - B) = G_{\nu}(A) + G_{\nu}(B)$,

(v) $G_{\mu}(kA) = |k| G_{\mu}(A)$,

(vi) $G_{\nu}(kA) = |k| G_{\nu}(A)$.

Proof. (i)

$$G_{\mu}(A + B) = (d_1 + d_2) - (a_1 + a_2)$$
$$- \frac{2((b_1 + b_2) - (a_1 + a_2)) + 2((d_1 + d_2) - (c_1 + c_2))}{\delta + 2}$$

$$= d_1 - a_1 - \frac{2(b_1 - a_1) + 2(d_1 - c_1)}{\delta + 2}$$

$$+ d_2 - a_2 - \frac{2(b_2 - a_2) + 2(d_2 - c_2)}{\delta + 2}$$

$$= G_\mu(A) + G_\mu(B).$$

(ii)

$$G_\mu(A - B) = (d_1 - a_2) - (a_1 - d_2)$$

$$- \frac{2((b_1 + c_2) - (a_1 + d_2)) + 2((d_1 - a_2) - (c_1 - b_2))}{\delta + 2}$$

$$= d_1 - a_1 - \frac{2(b_1 - a_1) + 2(d_1 - c_1)}{\delta + 2}$$

$$+ (-a_2) - (-d_2) - \frac{2(-c_2 - (-d_2)) + 2(-a_2 - (-b_2))}{\delta + 2}$$

$$= G_\mu(A) + G_\mu(B).$$

The proof is complete.

The proof of (iii) is similar to (i).

The proof of (iv) is similar to (ii).

(v) If $k > 0$

$$G_\mu(kA) = kd_1 - ka_1 - \frac{2(kb_1 - ka_1) + 2(kd_1 - kc_1)}{\delta + 2} = kG_\mu(A).$$

If $k < 0$

$$G_\mu(kA, \delta) = G_\mu(-|k|A, \delta) = |k|G_\mu(-A, \delta) = |k|G_\mu(A, \delta).$$

The proof of (vii) is similar to (v).

Theorem 3.3. *Let* $A = (a_1, a, b, c, d, d_1, 1, 0, \delta)$, *then*

(i) $a \leq V_\mu(A, \delta) \leq d,$

(ii) $a_1 \leq V_\nu(A, \delta) \leq d_1,$

(iii) $c - b \leq G_{\nu}(A, \delta) \leq d_1 - a_1,$

(iv) $c - b \leq G_{\mu}(A, \delta) \leq d - a.$

Proof. (i)

$$V_{\mu}(A, \delta) = \int_0^1 (L_1(\alpha) + U_1(\alpha))\alpha d\alpha = \int_0^1 (a + (b - a)\alpha^{\delta} + d - (d - c)\alpha^{\delta})\alpha d\alpha,$$

$$\geq \int_0^1 2a\alpha d\alpha = a,$$

and

$$V_{\mu}(A, \delta) = \int_0^1 (a + (b - a)\alpha^{\delta} + d - (d - c)\alpha^{\delta})\alpha d\alpha \leq \int_0^1 2d\alpha d\alpha = d.$$

Proof is complete.

(ii)

$$V_{\nu}(A, \delta) = \int_0^1 (L_2(\beta) + U_2(\beta))(1 - \beta)d\beta$$

$$= \int_0^1 (b(1 - \beta^{\delta}) + a_1\beta^{\delta} + c(1 - \beta^{\delta}) + d_1\beta^{\delta})(1 - \beta)d\beta,$$

$$= \int_0^1 (b - (b - a_1)\beta^{\delta} + c + (d_1 - c)\beta^{\delta})(1 - \beta)d\beta,$$

$$\geq \int_0^1 2a_1(1 - \beta)d\beta = a_1,$$

and

$$V_\nu(A,\, \delta) = \int_0^1 (b - (b - a_1)\beta^\delta + c + (d_1 - c)\beta^\delta)(1 - \beta)d\beta$$

$$\leq \int_0^1 2d_1(1 - \beta)d\beta = d_1.$$

Proof is complete.

Proof (iii) and (iv) are similar to (i) and (ii).

Remark 3.3. Let $A = (a_1', a_1, b_1, c_1, d_1, d_1', 1, 0, \delta)$ and $B = (a_2', a_2, b_2, c_2, d_2, d_2', 1, 0, \delta)$ be two GIFN_Bs; then for every δ_1 and δ_2, we have

(i) If $d_2 \leq a_1$, then $V_\mu(B,\, \delta_1) \leq V_\mu(A,\, \delta_2)$.

(ii) If $d_2' \leq a_1'$, then $V_\upsilon(B,\, \delta_1) \leq V_\nu(A,\, \delta_2)$.

(iii) If $d_2' - a_2' \leq c_1 - b_1$, then $G_\mu(B,\, \delta_1) \leq G_\mu(A,\, \delta_2)$.

(iv) If $d_1 - a_1 \leq d_1 - b_1$, then $G_\nu(B,\, \delta_1) \leq G_\nu(A,\, \delta_2)$.

Definition 3.3. Let $A = (a_1, a, b, c, d, d_1, \mu, \nu, \delta)$. A value index and an ambiguity index for the A are defined as follows:

$$V_\lambda(A,\, \delta) = V_\mu(A,\, \delta) + \lambda(V_\nu(A,\, \delta) - V_\mu(A,\, \delta)),$$

and

$$G_\lambda(A,\, \delta) = G_\nu(A,\, \delta) - \lambda\big(G_\nu(A,\, \delta) - G_\mu(A,\, \delta)\big),$$

respectively, where $\lambda \in [0, 1]$ is a weight which represents the decision maker's preference information. Limited to the above formulation, the choice $\lambda = 0.5$ appears to be reasonable one. One can choose λ according to the suitability of the subject. $\lambda = [0, 0.5]$ indicates decision maker's pessimistic attitude towards uncertainty while $\lambda = [0.5, 1]$ indicates decision maker's optimistic attitude towards uncertainty. With our choice $\lambda = 0.5$, the value and ambiguity index for GIFN_B reduces to:

$$V_{0.5}(A, \delta) = \frac{V_\mu(A, \delta) + V_\nu(A, \delta)}{2}, \quad G_{0.5}(A, \delta) = \frac{G_\mu(A, \delta) + G_\nu(A, \delta)}{2}.$$

Remark 3.4. Let $A = (a_1', a_1, b_1, c_1, d_1, d_1', 1, 0, \delta)$ and $B = (a_2', a_2, b_2, c_2, d_2, d_2', 1, 0, \delta)$ be two GIFN_Bs; then

(i) If $d_2' \leq a_1'$, then $V_\lambda(B, \delta) \leq V_\lambda(A, \delta)$.

(ii) If $d_2' - a_2' \leq c_1 - b_1$, then $G_\lambda(B, \delta) \leq G_\lambda(A, \delta)$.

Remark 3.5. If A is symmetric GIFN_B, then $V_\mu(A, \delta) = V_\nu(A, \delta) = V_{0.5}(A, \delta) = \frac{b + c}{2}$, and

$$G_\mu(A, \delta) = d - a - \frac{4(b - a)}{\delta + 2}, \quad G_\nu(A, \delta) = c - b - \frac{4(a_1 - b)}{(\delta + 1)(\delta + 2)},$$

$$G_{0.5}(A, \delta) = \frac{(c - b) + 2(b - a)}{2} - \frac{2(a_1 - b) + 2(\delta + 1)(b - a)}{(\delta + 1)(\delta + 2)}.$$

By using partial derivatives, we have

$$\frac{\partial G_{0.5}(A, \delta)}{\partial \delta} = \frac{((a_1 - b) + (c - d_1))(2\delta + 3) + ((b - a) + (d - c))(\delta + 1)^2}{(\delta + 1)^2(\delta + 2)^2}.$$

If A is symmetric GIFN_B, then

$$\frac{\partial G_{0.5}(A, \delta)}{\partial \delta} = \frac{(2(c - d_1))(2\delta + 3) + (2(d - c))(\delta + 1)^2}{(\delta + 1)^2(\delta + 2)^2}.$$

By using $\frac{\partial G_{0.5}(A, \delta)}{\partial \delta} = 0$, acceptable root is as $\delta^* = \frac{(d_1 - d) + \sqrt{(d_1 - d)^2 + (d - c)(3(d_1 - c) - (d - c))}}{d - c}$. Therefore ambiguity index is increasing for $\delta > \delta^*$, and is decreasing $\delta < \delta^*$. It is clear that $\delta^* > 1$, therefore ambiguity index is decreasing $\delta \leq 1$.

Definition 3.4. Let $A = (a_1', a_1, b_1, c_1, d_1, d_1', \mu_1, \nu_1, \delta)$ and $B = (a_2', a_2, b_2, c_2, d_2, d_2', \mu_2, \nu_2, \delta)$, then define ranking function for GIFN_B as follows:

$$R_\lambda(A, \delta) = V_\lambda(A, \delta) + G_\lambda(A, \delta),$$

where

(i) If $R(A, \delta) > R(B, \delta)$, then $A > B$.

(ii) If $R(A, \delta) < R(B, \delta)$, then $A < B$.

(iii) If $R(A, \delta) = R(B, \delta)$, then $A = B$.

If A is symmetric GIFN_B, then ranking function is increasing for $\delta > \delta^*$, and is decreasing $\delta < \delta^*$. Since $\delta^* > 1$, therefore ranking function is decreasing $\delta \leq 1$.

Theorem 3.4. *Let A and B be two $GIFN_B s$; then*

(i) $R(A + B, \delta) = R(A, \delta) + R(B, \delta)$,

(ii) $R(A - B, \delta) = R(A, \delta) + R(-B, \delta)$,

(iii) $R(kA, \delta) = kR(A, \delta), \quad k \in \mathbb{R}^+$,

(iv) $R(kA, \delta) = |k|R(-A, \delta), \quad k \in \mathbb{R}^-$.

Proof. (i) Since $V_\lambda(A + B, \delta) = V_\lambda(A, \delta) + V_\lambda(B, \delta)$ and $G_\lambda(A + B, \delta) = G_\lambda(A, \delta) + G_\lambda(B, \delta)$, then we have $R(A + B, \delta) = R(A, \delta) + R(B, \delta)$.

(ii) Since $V_\lambda(A - B, \delta) = V_\lambda(A, \delta) + V_\lambda(- B, \delta)$ and $G_\lambda(A - B, \delta) = G_\lambda(A, \delta) + G_\lambda(-B, \delta)$, then we have $R(A - B, \delta) = R(A, \delta) + R(-B, \delta)$.

(iii) Since $V_\lambda(kA, \delta) = kV_\lambda(A, \delta)$ and $G_\lambda(kA, \delta) = kG_\lambda(A, \delta)$, then we have $R(kA, \delta) = kR(A, \delta)$.

(iv) Since $V_\lambda(kA, \delta) = |k|V_\lambda(-A, \delta)$ and $G_\lambda(kA, \delta) = |k|G_\lambda(-A, \delta)$, then we have $R(kA, \delta) = |k|R(-A, \delta)$.

Remark 3.6. By using Theorem 4 (i) and (iii), we have

$$R(k_1 A + k_2 B, \delta) = k_1 R(A, \delta) + k_2 R(B, \delta),$$

where $k_1 > 0$ and $k_2 > 0$.

Theorem 3.5. *Let* A, B, C, *and* D *be four* $GIFN_B s$; *then*

(i) $A < B \Rightarrow A + C < B + C$,

(ii) $A < B, C < D \Rightarrow A + C < B + D$,

(iii) $A < B \Rightarrow kA < kB, k \in \mathbb{R}^+$.

Proof. Proof is obvious.

Example. Let $A_\delta = (0.25, 1, 2, 3, 4, 4.75, 1, 0, \delta)$,

$B = (5.25, 5.35, 5.45, 5.55, 5.65, 5.75, 1, 0, 2)$ be GIFN_Bs. Then for A_δ we have $\delta^* = 2.94$. According to Remark 3.5, we have $A_2 < A_1$, and $A_3 < A_4$. To further explore the indices were calculated.

$V_\mu(A, 2) = 2.5$, $V_\nu(A, 2) = 2.5$,

$G_\mu(A, 2) = 2$, $G_\nu(A, 2) = 1.58$, $V_{0.5}(A, 2) = 2.5$, $G_{0.5}(A, 2) = 1.79$, $R(A, 2) = 4.29$, $V_\mu(A, 1) = 2.5$, $V_\nu(A, 1) = 2.5$, $G_\mu(A, 1) = 1.67$, $G_\nu(A, 1) = 2.16$, $V_{0.5}(A, 1) = 2.5$, $G_{0.5}(A, 1) = 1.92$, $R(A, 1) = 4.42$.

Since $R(A, 2) < R(A, 1)$ it follows that $A_2 < A_1$.

$V_\mu(A, 3) = 2.5$, $V_\nu(A, 3) = 2.5$, $G_\mu(A, 3) = 2.2$, $G_\nu(A, 3) = 1.35$, $V_{0.5}(A, 3) = 2.5$, $G_{0.5}(A, 3) = 1.775$, $R(A, 3) = 4.275$, $V_\mu(A, 4) = 2.5$, $V_\nu(A, 4) = 2.5$, $G_\mu(A, 4) = 2.33$, $G_\nu(A, 4) = 1.23$, $V_{0.5}(A, 4) = 2.5$, $G_{0.5}(A, 4) = 1.78$, $R(A, 4) = 4.28$.

Since $R(A, 3) < R(A, 4)$ it follows that $A_3 < A_4$.

According to Remark 3.4, we have $V_{0.5}(A, 2) \leq V_{0.5}(B, 2)$, and $G_{0.5}(B, 2) \leq G_{0.5}(A, 2)$. To further explore the indices were calculated.

$$V_{\mu}(B, 2) = 5.5, \quad V_{\nu}(B, 2) = 5.5, \quad G_{\mu}(B, 2) = 0.2, \quad G_{\nu}(B, 2) = 0.167,$$

$$V_{0.5}(B, 2) = 5.5, \quad G_{0.5}(B, 2) = 0.1835.$$

Finally, we have $V_{0.5}(A, 2) \leq V_{0.5}(B, 2)$, and $G_{0.5}(B, 2) \leq G_{0.5}(A, 2)$.

4. Conclusion

In the present article, we studied two specifications of GIFN_B, namely, value and ambiguity, which are based on (α, β)-cut sets. They are used to define value index and ambiguity index. An algorithm for ranking of GIFN_Bs have been introduced in this paper which is based on these two indices.

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